

## Algebra 2 - UbD

### Probability and Statistics

#### Unit Overview

In this Algebra 2 Probability and Statistics unit, students will explore the difference between ideal predictions and realistic results, and the best ways to communicate these findings to an audience. In the first two weeks of this unit, students will become familiar with the ideas of simple probability and odds in order to predict the likelihood of future events. Students will utilize simple simulations using manipulatives (dice roll, card draw, coin flip) to explore compound probability events. These manipulatives will help differentiate between the concepts of experimental and theoretical outcomes. The third week of this unit will focus on the counting principle and identifying the total number of possibilities in which an event can occur by distinguishing the importance of order or lack thereof. Students will analyze real world datasets through measures of central tendency and determine meaningful representations of their findings. This unit will conclude with a project that utilizes student collected sleeping data over the course of this unit, which will be used to showcase students' understandings of statistical analyses and probabilistic predictions.

#### Goals:

- Differentiate between the concepts of experimental and theoretical probabilities
- Determine the importance, or unimportance, of order when calculating the probability of compound events through context clues
- Analyze data using measures of central tendency to communicate the meaning to an audience
- Create representations of important characteristics of a dataset

#### Enduring Understandings:

- A spectrum exists between absolute certainty, uncertainty, and impossibility.
- Predicted outcomes do not always directly translate to realistic outcomes.
- Patterns and regression trends can help determine the likelihood that an event occurs.
- Representations of data help to inform our current and future decisions.
- The choices we make to visually represent our data can positively or negatively impact others.

#### Essential Questions:

- How certain can we be that an event will definitely occur? Will never occur?
- What classifications make an object fair? biased?
- Why do we collect and analyze data?
- How is data read and interpreted to inform our future decisions?
- How is data utilized to influence others?
- To what extent can we rely on technological predictions to inform our future decisions?

<p><b>Students will be able to...</b></p> <ul style="list-style-type: none"> <li>● Differentiate between theoretical and empirical outcomes.</li> <li>● Use fair objects to calculate simple probabilities of singular and compound events.</li> <li>● Distinguish permutations and combinations using factorials and their formulas.</li> <li>● Measure central tendency of a dataset (mean, median, mode, range, IQR) and select the best representation to communicate their data.</li> </ul>	<p><b>Students will know . . .</b></p> <ul style="list-style-type: none"> <li>● How to construct a sample space and determine the probability that an event will or will not occur.</li> <li>● How to calculate probabilities of simple, singular events and compound events.</li> <li>● How to calculate measures of central tendency (mean, median, mode, range, IQR).</li> <li>● How to best communicate the trends and patterns in a dataset.</li> </ul>
<p><b>Assessment Evidence:</b></p> <ul style="list-style-type: none"> <li>● Standards-based quizzes (2) assessing specific content</li> <li>● (Benchmark) project: using probability and statistics to analyze individual sleep data with a partner (using app or written sleep log) - Explained at end of Unit Plan</li> </ul>	<p><b>Performance Tasks:</b></p> <ul style="list-style-type: none"> <li>● Daily presentation of Warm-Up problems</li> <li>● Daily Homework Assignments</li> </ul>

## Pennsylvania State Standards

Standard	Description
CC.2.4.HS.B.1	Summarize, represent, and interpret data on a single count or measurement variable.
CC.2.4.HS.B.2	Summarize, represent, and interpret data on two categorical and quantitative variables.
CC.2.4.HS.B.3	Analyze linear models to make interpretations based on the data.
CC.2.4.HS.B.4	Recognize and evaluate random processes underlying statistical experiments.
CC.2.4.HS.B.5	Make inferences and justify conclusions based on sample surveys, experiments, and observational studies.
CC.2.4.HS.B.6	Use the concepts of independence and conditional probability to interpret data.
CC.2.4.HS.B.7	Apply the rules of probability to compute probabilities of compound events in a uniform probability model.

## Outline of Week

<b>Week 1</b>	<ul style="list-style-type: none"> <li>● Introduce Probability and Odds</li> <li>● Differentiate between Independent, Dependent, and Conditional Events</li> </ul>
<b>Week 2</b>	<ul style="list-style-type: none"> <li>● Investigate Applications of Probability</li> <li>● Standards Quiz: Probability</li> </ul>
<b>Week 3</b>	<ul style="list-style-type: none"> <li>● Introduce Counting, Permutations, and Combinations</li> <li>● Understand the components of Central Tendency and Visual Representations of Data</li> <li>● Investigate Applications of Central Tendencies</li> </ul>
<b>Week 4</b>	<ul style="list-style-type: none"> <li>● Standards Quiz: Permutations, Combinations, Central Tendency</li> <li>● Project - Data Analysis: Sleep</li> </ul>
<b>Week 5</b>	<ul style="list-style-type: none"> <li>● Project - Data Analysis: Sleep</li> <li>● Project Presentations</li> </ul>

## Daily Lessons and Learning Activities

<b>Day 1 (W1, D1)</b>	<p><b>Warm-Up (Small Groups):</b> Coin Flip Prediction Activity</p> <p><b>Body (Whole Group):</b></p> <ul style="list-style-type: none"> <li>● What is probability? What are odds?</li> <li>● Theoretical vs. Experimental Probability</li> <li>● Intersections, Unions, Complements</li> <li>● Independent vs. Dependent vs. Mutually Exclusive Events</li> <li>● Introduce Project Sleep Log (See Project Outline Below) <ul style="list-style-type: none"> <li>○ Categories to track, how often to track</li> </ul> </li> </ul> <p><b>Wrap-Up:</b></p> <ul style="list-style-type: none"> <li>● How does theoretical probability differ from experimental probability?</li> <li>● What factors influence theoretical and experimental probabilities? Can we make them the same?</li> <li>● How is the probability of an event occurring different than the odds?</li> </ul> <p><b>HW:</b> First Night of Sleep Log</p>
<b>Day 2 (W1, D2)</b>	<p><b>Warm-Up:</b> Sleep Log Check; Hunger Games probability scenarios</p> <p><b>Body (Small Groups):</b></p> <ul style="list-style-type: none"> <li>● Playing Cards Probability Exploration</li> <li>● Playing Cards Probability Exploration Review</li> </ul> <p><b>Wrap-Up:</b></p> <ul style="list-style-type: none"> <li>● How do independent, dependent, and mutually exclusive events differ?</li> <li>● What is a union? What is an intersection?</li> </ul> <p><b>HW:</b> Game Show video clips + Questions; Sleep Log</p>
<b>Day 3 (W1, D3)</b>	<p><b>Warm-Up:</b> Sleep Log Check; Playing cards probability scenarios</p> <p><b>Body (Whole + Small Group):</b></p> <ul style="list-style-type: none"> <li>● Egg Roulette video</li> <li>● Egg Roulette probability calculation</li> <li>● Egg Roulette Simulation</li> <li>● Conditional probability Intro</li> </ul>

	<p><b>Wrap-Up:</b></p> <ul style="list-style-type: none"> <li>● How could conditional probability be seen in Egg Roulette?</li> <li>● What other game shows did you see conditional probability exist?</li> </ul> <p><b>HW:</b> Create + solve two original probability problems; Sleep Log</p> <ul style="list-style-type: none"> <li>● 1st prob: independent probability</li> <li>● 2nd prob: dependent probability</li> </ul>
<p><b>Day 4 (W1, D4)</b></p>	<p><b>Warm-Up:</b> Sleep Log Check; Switch HW problems with partner + solve</p> <p><b>Body (Whole Group):</b></p> <ul style="list-style-type: none"> <li>● Conditional probability vs. Dependent probability</li> <li>● Conditional probability examples</li> <li>● Changing dependent probability problems from HW into conditional</li> </ul> <p><b>Wrap-Up:</b></p> <ul style="list-style-type: none"> <li>● What does it mean for events to be conditional?</li> <li>● How is dependent probability related to conditional probability</li> </ul> <p><b>HW:</b> Mixed Probability Practice; Sleep Log</p>
<p><b>Day 5 (W2, D1)</b></p>	<p><b>Warm-Up:</b> Sleep Log Check; Monty Hall Problem</p> <p><b>Body (Small Groups Rotating):</b></p> <ul style="list-style-type: none"> <li>● HW Review</li> <li>● Probability Mixed Review Stations <ul style="list-style-type: none"> <li>○ Birthday Problem</li> <li>○ Dice Probability Questions</li> <li>○ Blackjack Probability</li> <li>○ Go Fish Probability</li> </ul> </li> </ul> <p><b>Wrap-Up:</b></p> <ul style="list-style-type: none"> <li>● How likely is it that two people (students + teacher + SAT) in our classroom share the same birthday?</li> <li>● Where else does probability appear at SLA? In our everyday lives?</li> </ul> <p><b>HW:</b> Research a current event article of your choosing + develop a meaningful prediction based on the given information (Presidential Election; sports event); Sleep Log</p>
<p><b>Day 6 (W2, D2)</b></p>	<p><b>Warm-Up:</b> Sleep Log Check; With a partner from a different table,</p> <ul style="list-style-type: none"> <li>● Briefly summarize your article</li> <li>● Share any important information</li> <li>● Your partner will solve your probability problem</li> <li>● Check your calculations together</li> </ul> <p><b>Body (Small Groups or Individual):</b></p> <ul style="list-style-type: none"> <li>● Basketball probability application: Mathalicious Online Resource (modified) <ul style="list-style-type: none"> <li>○ Question: If your team is winning by 2 points, should you foul the other team on a possible game winning three-point shot?</li> <li>○ Using season free throw percentages to predict game outcome based on making 3, 2, 1, or no shots.</li> <li>○ Calculating the probability of winning, losing, overtime from offensive and defensive perspectives <ul style="list-style-type: none"> <li>■ Connection: offense winning is defense losing</li> </ul> </li> <li>○ Extension Task - Q6 creating an inequality to model when the defense should foul on a 3-point shot</li> </ul> </li> </ul> <p><b>Wrap-Up:</b></p> <ul style="list-style-type: none"> <li>● How accurate is a player's season free throw percentage?</li> <li>● Overall, would you recommend fouling in a game making/breaking position?</li> </ul>

	<p>Why or why not?</p> <p><b>HW:</b> Complete Basketball Probability Application and Reflection Questions; Sleep Log</p>
<p><b>Day 7</b> <b>(W2, D3)</b></p>	<p><b>Warm-Up:</b> Sleep Log Check; How are independent, dependent, and conditional events different? Provide an example of each.</p> <p><b>Body (Whole Group):</b></p> <ul style="list-style-type: none"> <li>● HW Review: Extension Questions</li> <li>● Standards Review: Kahoot!</li> </ul> <p><b>Wrap-Up:</b></p> <ul style="list-style-type: none"> <li>● How can you use context to decide the type of event that occurs?</li> <li>● How are theoretical and experimental probability different?</li> <li>● What can an experimenter do to reduce the difference?</li> </ul> <p><b>HW:</b> Mock Quiz 1; Sleep Log</p>
<p><b>Day 8</b> <b>(W2, D4)</b></p>	<p><b>Warm-Up:</b> Sleep Log Check; Standards Quiz Preparation Reflection</p> <ul style="list-style-type: none"> <li>● Which standard(s) are you the most confident with?</li> <li>● What methods did you use to prepare?</li> <li>● How could have you prepared differently?</li> </ul> <p><b>Body (Individual):</b></p> <ul style="list-style-type: none"> <li>● Standards Quiz - Probability <ul style="list-style-type: none"> <li>○ Probability vs. Odds</li> <li>○ Independent, Dependent, Mutually Exclusive Events</li> <li>○ Conditional Probability</li> </ul> </li> </ul> <p><b>Wrap-Up:</b> None</p> <p><b>HW:</b> Sleep Log</p>
<p><b>Day 9</b> <b>(W3, D1)</b></p>	<p><b>Warm-Up:</b> Sleep Log Check; Possibilities problem - 3 shirts, 3 pants, 2 socks</p> <p><b>Body (Whole Group):</b></p> <ul style="list-style-type: none"> <li>● Counting Principle</li> <li>● Factorials, Permutations, Combinations</li> <li>● Chair arrangement demo</li> </ul> <p><b>Wrap-Up:</b></p> <ul style="list-style-type: none"> <li>● When do we use permutations vs. combinations?</li> <li>● How do we know when order does or does not matter?</li> </ul> <p><b>HW:</b> Counting, Permutations, Combinations worksheet; Sleep Log</p>
<p><b>Day 10</b> <b>(W3, D2)</b></p>	<p><b>Warm-Up:</b> Sleep Log Check; Six Space License Plate problem</p> <p><b>Body (Whole Group + Small Group):</b></p> <ul style="list-style-type: none"> <li>● HW Review</li> <li>● Permutations + Combinations Continued</li> <li>● Permutations + Combinations practice <ul style="list-style-type: none"> <li>○ Creating original problems by table - permutation, combination, counting <ul style="list-style-type: none"> <li>■ Extension: connecting probability with permutations and/or combinations</li> </ul> </li> <li>○ Trading with another table to solve</li> <li>○ Reflection: What types of problems gave you trouble? What similarities existed if any?</li> </ul> </li> </ul> <p><b>Wrap-Up:</b></p> <ul style="list-style-type: none"> <li>● What context clues exist for problems using permutation, combination, or counting?</li> </ul>

	<ul style="list-style-type: none"> <li>● How might probability be used with permutations and combinations?</li> </ul> <p><b>HW:</b> Probability, Permutations, + Combinations Linked ; Sleep Log</p>
<p><b>Day 11</b> <b>(W3, D3)</b></p>	<p><b>Warm-Up:</b> Sleep Log Check; What is the probability of winning a 4-number lottery when all numbers are chosen at once? (Winning means having all numbers in any order)</p> <p><b>Body:</b></p> <ul style="list-style-type: none"> <li>● HW Review</li> <li>● Measures of Central Tendency (mean, median, mode, min, max, range, IQR)</li> <li>● Visual Representations: Box-and-Whisker Plot, Stem-and-Leaf Plot Scatterplot, Bar Graphs</li> <li>● Trend lines, Line of best fit, Correlation</li> </ul> <p><b>Wrap-Up:</b></p> <ul style="list-style-type: none"> <li>● How can various measures of central tendency convey more meaning from our data?</li> <li>● How can the mode of displaying data convey more (or less) meaning?</li> </ul> <p><b>HW:</b> M.o.C.T., Graphs, and Line of Best Fit Practice; Sleep Log</p>
<p><b>Day 12</b> <b>(W3, D4)</b></p>	<p><b>Warm-Up:</b> Sleep Log Check; Complete the Google form for Age, Height, School Arrival Time</p> <p><b>Body (Small Group Investigation):</b></p> <ul style="list-style-type: none"> <li>● Group Data Analysis <ul style="list-style-type: none"> <li>○ Movie ticket prices over time <ul style="list-style-type: none"> <li>■ Percent change</li> <li>■ Measures of Central Tendency</li> <li>■ Visual Representation (1)</li> </ul> </li> </ul> </li> </ul> <p><b>Wrap-Up:</b></p> <ul style="list-style-type: none"> <li>● How are the mean and median influenced in a data set? Are they the same?</li> <li>● What is the most valuable measure of central tendency?</li> </ul> <p><b>HW:</b> Use the Class Data to find M.o.C.T. and create 1 meaningful visual; Sleep Log</p>
<p><b>Day 13</b> <b>(W4, D1)</b></p>	<p><b>Warm-Up:</b> Sleep Log Check; Two visual representations of data -- why are they misleading?</p> <p><b>Body:</b></p> <ul style="list-style-type: none"> <li>● Review class visuals</li> <li>● Standards Review: Kahoot!</li> </ul> <p><b>Wrap-Up:</b></p> <p><b>HW:</b> Mock Quiz 2; Sleep Log</p>
<p><b>Day 14</b> <b>(W4, D2)</b></p>	<p><b>Warm-Up:</b> Sleep Log Check; Standards Quiz Preparation Reflection</p> <ul style="list-style-type: none"> <li>● Which standard(s) are you the most confident with?</li> <li>● What methods did you use to prepare?</li> <li>● How could have you prepared differently?</li> </ul> <p><b>Body:</b></p> <ul style="list-style-type: none"> <li>● Standards Quiz 2 <ul style="list-style-type: none"> <li>○ Counting Principle</li> <li>○ Permutations</li> <li>○ Combinations</li> <li>○ Measures of Central Tendency</li> <li>○ Modeling Data</li> </ul> </li> </ul> <p><b>Wrap-Up:</b> None</p> <p><b>HW:</b> Last Day Sleep Log</p>

<p><b>Day 15</b> <b>(W4, D3)</b></p>	<p><b>Warm-Up:</b> Sleep Log Check; Why might collecting personal sleep data be useful?</p> <p><b>Body:</b></p> <ul style="list-style-type: none"> <li>● Introduce Sleep Project</li> <li>● Goals for class: <ul style="list-style-type: none"> <li>○ Select partner</li> <li>○ Compile data in Google Sheet</li> </ul> </li> </ul> <p><b>Wrap-Up:</b></p> <ul style="list-style-type: none"> <li>● How might sleep data be informative in your daily life?</li> <li>● Who might benefit from collecting sleep data?</li> </ul> <p><b>HW:</b> Project Task due next class</p>
<p><b>Day 16</b> <b>(W4, D4)</b></p>	<p><b>Warm-Up:</b> Outliers Discussion</p> <ul style="list-style-type: none"> <li>● Do you and your partner notice any outliers in your data?</li> <li>● What influence can outliers have on interpreting your data?</li> </ul> <p><b>Body:</b></p> <ul style="list-style-type: none"> <li>● Project Work Time</li> <li>● Goals for class: <ul style="list-style-type: none"> <li>○ Find Measures of Central Tendency</li> <li>○ Create at least 2 meaningful visual representations</li> <li>○ Create at least 2 probability and/or odds predictions for future</li> </ul> </li> </ul> <p><b>Wrap-Up:</b></p> <ul style="list-style-type: none"> <li>● What makes a graph, chart, visual “meaningful”?</li> <li>● What properties of a visual must you consider such that it is “truthful”?</li> </ul> <p><b>HW:</b> Project Task due end of next class</p>
<p><b>Day 17</b> <b>(W5, D1)</b></p>	<p><b>Warm-Up:</b> Provide class with my own sleep data, each table should develop:</p> <ul style="list-style-type: none"> <li>● 1 probability/odds statement based on patterns seen</li> <li>● 1 measure of central tendency statement</li> <li>● 1 question about further information they might need</li> </ul> <p><b>Body:</b></p> <ul style="list-style-type: none"> <li>● Project Work Time</li> <li>● Goals for class: <ul style="list-style-type: none"> <li>○ Finalize visual representations</li> <li>○ Finalize probability and/or odds predictions</li> <li>○ Begin creating presentation tool</li> </ul> </li> </ul> <p><b>Wrap-Up:</b></p> <ul style="list-style-type: none"> <li>● Based on the sleep information we collected this unit, what limitations exist in our predictions?</li> <li>● What other information might be useful to inform others about sleep?</li> </ul> <p><b>HW:</b> Project Task due tomorrow</p>
<p><b>Day 18</b> <b>(W5, D2)</b></p>	<p><b>Warm-Up:</b> No warm-up to allow for full work period</p> <p><b>Body:</b></p> <ul style="list-style-type: none"> <li>● Project Work Time</li> <li>● Goals for class: Finalize Project + Presentation <ul style="list-style-type: none"> <li>○ Create Slideshow, Brochure, Advertisement, Testimonials, Infographic</li> <li>○ Additionally, create poster or flyers to put around school</li> </ul> </li> </ul> <p><b>Wrap-Up:</b></p> <ul style="list-style-type: none"> <li>● What are some components of meaningful data representation?</li> <li>● How can audiences be “tricked” by data representations?</li> </ul> <p><b>HW:</b> Prepare Final Project + Presentation</p>

**Day 19  
(W5, D3)**

**Warm-Up:** None due to Presentations

**Body:**

- Group Presentations (Half of Groups)
  - Audience will have to provide two peaks and two valleys for each presenting group (ask questions, provide suggestions for future analysis or observation)

**Wrap-Up:**

- Were there any consistent patterns amongst groups?
- What was one interesting pattern that you heard?

**HW:** Project Reflection (Presenters)

- For each rubric category (Design, Knowledge, Application, Process, and Presentation), give yourself a grade and explain why you feel you earned it.
- Which category do you believe you excelled in and why?
- If you were given a chance to revise your final project based on the feedback from your peers, what would you improve?
- What further questions would you like to explore based on your analysis and conclusions?

**Day 20  
(W5, D4)**

**Warm-Up:**

- What impact, if any, has your study and analysis had on your sleep?
- What changes can you foresee yourself making in the future?

**Body:**

- Group Presentations (Half of Groups)
  - Audience will have to provide two peaks and two valleys for each presenting group (ask questions, provide suggestions for future analysis or observation)

**Wrap-Up:**

- Were there any consistent patterns amongst groups?
- What was one interesting pattern that you heard?

**HW:** Project Reflection (Presenters)

- For each rubric category (Design, Knowledge, Application, Process, and Presentation), give yourself a grade and explain why you feel you earned it.
- Which category do you believe you excelled in and why?
- If you were given a chance to revise your final project based on the feedback from your peers, what would you improve?
- What further questions would you like to explore based on your analysis and conclusions?

## (Benchmark) Project: Data Analysis - Sleep

Math	Science
<p>Questions:</p> <ol style="list-style-type: none"> <li>1. What patterns can we see from our own sleep?</li> <li>2. How can these patterns inform our future?</li> <li>3. What outside variables impact our sleep?</li> </ol> <ul style="list-style-type: none"> <li>● Students track sleep over the course of 2-3 weeks               <ul style="list-style-type: none"> <li>○ Using written sleep log or free phone app</li> <li>○ Track: time sleeping, quality of sleep rating, day of the week, weather outside, number of assignments due next day</li> </ul> </li> <li>● Find all measures of central tendency (mean, median, mode, range, IQR)</li> <li>● Create at least 2 meaningful visual representations of individual data</li> <li>● Use at least 2 statements of probability and/or odds to predict future sleep patterns</li> <li>● Analyze alteration to sleep practices (from Science) and compare patterns and trends</li> </ul>	<p>Questions:</p> <ol style="list-style-type: none"> <li>1. Why is sleep important?</li> <li>2. Is our collected data valid?</li> <li>3. If I alter my sleep “practices” will I improve or worsen my sleep pattern?</li> </ol> <ul style="list-style-type: none"> <li>● Students research impact of sleep               <ul style="list-style-type: none"> <li>○ Rooted in age, gender, race, physical and mental demand</li> <li>○ Sleep cycles, REM sleep, non-REM sleep</li> <li>○ “Ideal” amount of sleep vs. oversleeping vs. sleep deprivation</li> </ul> </li> <li>● Validity of Math analysis</li> <li>● Experiment with a change in sleep practices to observe impact on sleep (independent, dependent, control variables)               <ul style="list-style-type: none"> <li>○ Not use technology 1 hour before going to sleep</li> <li>○ Reading 30 minutes before going to sleep</li> <li>○ Sleeping with earplugs (silence) or listening to music</li> </ul> </li> </ul>

